Robust Performance Limitations and Design of Controlled Delayed Systems

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This paper presents performance limitations and a control design methodology for nonminimum phase plants of the pure delay type subject to robustness constraints. Of interest is the design of a set of controllers, for which the open-loop transfer function is a proportional-integral (PI) controller plus delay, meeting constraints on the magnitude of the closed-loop transfer function and on the plant gain uncertainty. These two specifications are used to characterize the robustness, and are a recommended alternative to the gain and phase margin constraints. A control design plot is presented which allows for selection of controller parameters including those for the lowest sensitivity controller, and graphically highlights gain and phase margin tradeoffs. The paper discusses limitations of performance of such systems in terms of crossover frequency and sensitivity. In addition, expressions and design plots are provided for a simplified approximate solution. [DOI: 10.1115/1.1849246]

Keywords: Linear Systems, Nonminimum Phase, Delay, Gain Margin, Phase Margin

1 Introduction

For a nonminimum-phase (NMP) plant, such as a plant with pure delay, classical proportional-integral derivative (PID) controller tuning methods—such as the Ziegler–Nichols [1] and Cohen–Coon [2] methods—may not be appropriate for achieving the requisite performance [3]. These methods become especially problematic when designing a controller for a plant with a delay that is large relative to its time constant. Despite problems, classical PI or PID control tuning methods are often used but "detuned" to maintain overall stability [4].

Many plants do not have fixed models and/or exhibit variable delay. A traditional PID controller may show performance degradation or may even become unstable as plant parameters change. Adaptive controllers are often implemented to accommodate these changing situations. Methods include generalized predictive control (GPC), relay feedback autotuning, internal model control (IMC), and the adaptive Smith predictor method [5,6].

The design of classical controllers for delayed systems has been the subject of much interest [7–9]. Khan and Lehman [10] developed PI control algorithms for first-order plants with time delay. They performed extensive simulations and data fittings to obtain PI tuning formulas (for ratios of time delay to time constant ranging from 0.2 to 20). Alexander and Tahan [11] compared a tuning method proposed by Abbas [12] to an adaptive Smith predictor control strategy for control of systems with time delay.

Lee et al. [13] presented a method for PID controller tuning based on process models for integrating and unstable processes with time delay. They provided explicit PID controller tuning rules for unstable plants with one right-half-plane (RHP) pole, delayed unstable plants with two RHP poles, and integrating plants with time delay. Mann et al. [14] conducted a time-domain PID analysis that included three types of first-order plus time delay models (zero or negligible, low to medium long, and very long time delays).

Methods for tuning PID controllers based on gain and phase margin specifications have been reported. Ho et al. [15] developed simple analytical formulas to design PI and PID controllers for commonly used first- and second-order plus dead time plant models to meet gain and phase margin specifications. Ho et al. [16] presented tuning formulas for the design of PID controllers that satisfy both robustness and performance requirements.

Specific methods have been developed for the design of PI controllers. Astrom et al. [17] described a numerical method for designing PI controllers based on optimization of load disturbance rejection with constraints on sensitivity and weighting of set point response. They suggested, as did Horowitz and Sidi [18], the use of the maximum sensitivity as an important practical design specification.

Sidi [19] and Horowitz and Sidi [20] presented an optimal robust synthesis technique to design a feedback controller for an uncertain NMP plant that can achieve specified closed-loop performance. Their synthesis technique provides the designer with insight into the tradeoffs between closed-loop performance and bandwidth, and also defines an implicit criterion for determining whether a solution exists. Sidi [21] developed a criterion to estimate the maximum bandwidth of a sampled plant for given gain and phase margin. He assumed an open-loop form of the ideal Bode characteristics and used asymptotic approximations. Horowitz and Liau [22] extended this technique to stable plants with several RHP zeros. They showed how to achieve a large open-loop gain in several frequency ranges, although there would always be some frequency ranges determined by the RHP zeros, in which the open-loop gain would be less than 0 dB. This fact was proven by Francis and Zames [23] and by Freudenberg and Looze [24] who showed that for NMP plants, a small sensitivity in one frequency range forces a large sensitivity in the complementary range. Freudenberg and Looze [24,25] developed several constraints on the closed-loop sensitivity of NMP and/or unstable plants in the form of weighted integrals of the sensitivity at all frequencies or over a frequency range where the open-loop gain is much less than one. Middleton [26] used their results to provide a bandwidth limitation on NMP and/or unstable plants. Crossover frequency limitations assuming a given slope of the open-loop amplitude around the crossover frequency were given in [27].

Seron et al. [28] addressed fundamental limitations in control system design. As indicated in the Preface of their textbook, limitations in control are core issues of feedback theory and govern what is achievable, and conversely what is not achievable, in feedback systems. The subject has a rich history, beginning with the seminal work of Bode [29]. Interpretations of Bode's results in the context of control system design were provided by Horowitz [30].

1.1 Scope. This paper studies limitations of systems with pure delay subjected to constraints on the upper bound of the complementary sensitivity (i.e., magnitude of the closed-loop transfer function) and the plant gain uncertainty. These two constraints provide a broader measure of robustness than the gain margin and phase margin. Explicit equations are provided that highlight the tradeoff among the open-loop crossover frequency, sensitivity, and high-frequency gain. The paper also includes a design technique that provides parameters for a specific controller for a large class of practical systems. The structure considered for

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the open-loop transfer function is PI plus integrator plus delay. The results of the design technique are captured in a single plot that provides the set of all controllers that satisfy the specifications on the sensitivity upper bound, gain margin, and phase margin, and graphically illustrates performance tradeoffs among them.

2 Problem Statement

Robustness is represented via the complementary sensitivity bound, M, and the gain uncertainty of the plant in an interval $[1,K]$, both of which are assumed known. These two parameters are embedded in the following inequality for the magnitude of the closed-loop transfer function,

$$\left|\frac{kL(s)}{1+kL(s)}\right| \leq M \quad \text{for } s=j\omega,\ \forall \omega \geq 0,\ k\in[1,K], \tag{1}$$

where the complementary sensitivity bound $M>1$, the gain uncertainty of the plant k is in the interval $[1,K]$, and the open-loop transfer function, $L(s)$, is

$$L(s)=C(s)P(s)e^{-sT}, \tag{2}$$

where $C(s)$ is the controller and $P(s)e^{-sT}$ is the plant, with $P(s)$ being minimum phase and stable.

Inequality (1) is a more encompassing measure of robustness than the gain margin and phase margin [18]. It places a bound on the sensitivity at all frequencies, not just at the two crossover frequencies associated with the gain and phase margins. As an example demonstrating the limitation of gain and phase margins to fully capture robustness, consider the following open-loop transfer function,

$$L(s)=k\frac{1.39s+211}{s}\frac{100}{s(1+s/1000)}e^{-0.0004s}, \quad k\in[1,10]$$

whose phase and gain margins are, respectively, 35 deg and 24 dB for $k=1$. These values are the minimal margin values corresponding to $M=1.66$ and $K=10$ in (1). The M and K specifications guarantee that the phase margin is at least 35° for any gain, k, in the interval $[1,10]$. However, from a frequency-domain analysis of $L(s)$, the phase margin for $k=10$ is calculated to be 13° and the gain margin is 4 dB. Thus, the margin specifications fail to guarantee the satisfaction of the phase margin for all plant gain uncertainties. In its place, the complementary sensitivity parameter M and the gain uncertainty K are recommended, as these parameters guarantee satisfaction of the phase margin for all possible gain uncertainties.

It can be shown [31] that M and K are related to the gain and phase margins. For example, when $\arg L(j\omega)=-\pi$ rad, then (1) requires $|L(j\omega)|\leq(M-1)/M$ for $K=1$, and thus the gain margin, GM, for a given K is at least

$$\text{GM}=20\log_{10}(K)+20\log_{10}\left(\frac{M+1}{M}\right). \tag{3}$$

Similarly, when $|L(j\omega)|=1$, (1) requires $\arg L(j\omega)>-\pi+2\arcsin[(2M)^{-1}]$, and thus the phase margin, PM, is at least

$$\text{PM}=2\arcsin\left(\frac{1}{2M}\right). \tag{4}$$

2.1 Design Challenge. This paper considers the set of controllers of the special form,

$$C(s)=\frac{A(1+Bs)}{s^2P(s)}, \tag{5}$$

corresponding to the open-loop transfer function,

$$L(s)=\frac{A(1+Bs)}{s^2}e^{-sT}. \tag{6}$$

The transfer function of (6) can be interpreted as the product of an integrator, a PI controller with integral gain A and proportional gain AB, and delay e^{-sT}. At low frequencies the transfer function is proportional to A/s^2, and thus the inverse of the integral gain A is proportional to the sensitivity. The gain AB is proportional to the sensor noise effect at the plant input.

The issue of properness of $C(s)$ is not considered here. One can always add poles far enough from the origin to ensure $C(s)$ is a proper controller. Furthermore, the high-frequency dynamics of $P(s)$ can be replaced by a pure delay, as is often done in process control, making $C(s)$ proper.

The design problem of interest is to find all (A,B) pairs that satisfy (1) and, in particular, to find the pair for which A is maximum, corresponding to the controller with lowest sensitivity at low frequencies. The problem is solved exactly and, in addition, simplified approximate solutions are derived. For the latter, $L(s)$ is replaced by the ZOH of $A(1+Bs)/s^2$ with sampling time $T_s=\alpha T$ (with the chosen value of α explained later). It is shown that the solution of the approximate problem matches closely the solution of the exact problem.

The motivation for using the open-loop transfer function of the form (6), or its ZOH with sampling time T_s for the approximate case, is that it is a very reasonable model of open-loop low-frequency behavior of many practical systems, including motor speed controllers and phase-lock-loops (PLLs). Although the method developed below is applicable to many real-world systems, the proposed controller structure is not recommended for two particular types of systems:

1. It is not recommended that systems with lightly damped frequencies for which the open-loop amplitude is significantly above 0 dB be "notched" by $1/P(s)$ because this will decrease the disturbance rejection and increase the sensitivity.
2. It is not recommended that systems exhibiting notchlike behavior (antiresonance) be amplified by $1/P(s)$. The reason is that an antiresonance most likely is associated with resonance at a different location of the plant, which if amplified by $1/P(s)$ might dangerously excite the plant.

3 Main Results

3.1 Exact Solution. To find (A,B) values representing the solutions of (1) where L is given by (6), introduce the change of variables,

$$a=AT^2, \quad b=\frac{B}{T}, \quad \Omega=\omega T, \tag{7}$$

and consider first the case of no gain uncertainty, i.e., $K=1$. It follows from (1) that the (a,b) pairs must satisfy the inequality,

$$M^2\Omega^4-2M^2ab\sin(\Omega)\Omega^3+[a^2b^2(M^2-1)-2aM^2\cos(\Omega)]\Omega^2 + [a^2(M^2-1)]\geq 0 \tag{8}$$

for all normalized frequencies $\Omega\geq 0$. For an (a,b) pair which is on the boundary region of the allowed (a,b) values, Ω exists such that (8) is an equality. In addition, the derivative of the left-hand side of (8) with respect to Ω at the same frequency is zero,

$$2M^2[2-ab\cos(\Omega)]\Omega^2+2M^2a(1-3b)\sin(\Omega)\Omega - 4M^2a\cos(\Omega)+2a^2b^2(M^2-1)=0. \tag{9}$$

Subtracting four times the equality of (8) from (9) yields the following expression for a, at that Ω,

$$a=-\frac{\Omega^2M^2[(b\Omega^2-2)\cos(\Omega)-\Omega(b+1)\sin(\Omega)]}{(b^2\Omega^2+2)(M^2-1)}. \tag{10}$$

Substituting (10) into the equality of (8) gives a fourth-order equation for b as a function of Ω,

$$x_4b^4+x_3b^3+x_2b^2+x_1b+x_0=0, \tag{11}$$

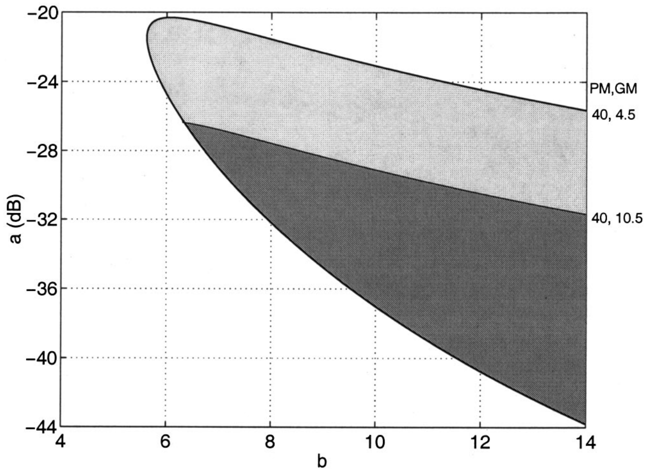

Fig. 1 Region of (a,b) values for $M=1.46$, equivalent to at least 40 deg phase margin (PM) and at least 4.5 dB gain margin (GM) for $K=1$ (both shaded regions). Lower shaded region is for $M=1.46$ with additional 6 dB plant gain uncertainty ($K=2$).

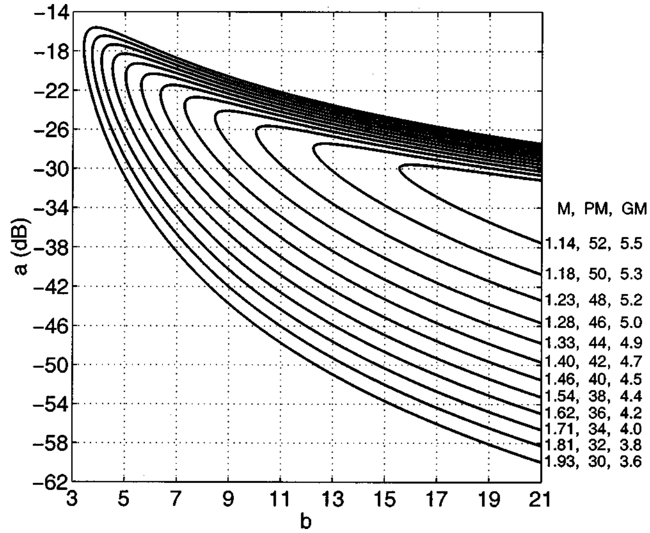

Fig. 2 Boundary curves of (a,b) values that satisfy (1) with $K=1$ for L of (6) and change of variables (7). Marked on the right of each curve is its M value and corresponding lower bound of phase margin (PM) and gain margin (GM in dB) for $K=1$, according to (3) and (4).

where[1]

$$x_4=\Omega^4[1-M^2(\Omega^2+1)\cos^2(\Omega)]$$

$$x_3=2M^2\Omega^3\cos(\Omega)[(\Omega^2+1)\sin(\Omega)+\Omega\cos(\Omega)]$$

$$x_2=\Omega^2[-M^2((4\Omega\sin(\Omega)+3\cos(\Omega))\cos(\Omega)+\Omega^2+1)+4]$$

$$x_1=2M^2\Omega\sin(\Omega)[\Omega\sin(\Omega)+(\Omega^2+4)\cos(\Omega)]$$

$$x_0=4-M^2(\Omega^2+4)\sin^2(\Omega).$$

To interpret the solution of (11) consider the particular case of $M=1.46$, which is equivalent to a phase margin of at least 40 deg and gain margin of at least 4.5 dB assuming no gain uncertainty ($K=1$). The solution corresponds to a two-dimensional region in the (a,b) plane, which can be calculated as follows: use (11) to solve for b for a given Ω. Noting that b (for that Ω) has four solutions, select the positive real one for which the resulting open-loop system is stable. Then calculate a from (10). Searching over a range of frequencies Ω enables the boundary of the (a,b) region that satisfies (8) to be identified, as depicted in Fig. 1.

Now assume the plant suffers from gain uncertainty, given by its upper bound K. Figure 1 can also be used to find the (a,b) region that satisfies M and K specifications (due to M) and plant gain uncertainty (due to K). For example, to account for a gain uncertainty of 6 dB (that is, $K=2$), then for any b, the allowed a values should be 6 dB less in order to cope with the plant gain uncertainty $k\in[1,2]$. The (a,b) region will therefore be the lower shaded region shown in Fig. 1 where the upper curve is shifted down by 6 dB.

The (a,b) pairs on the boundary are values for which there is at least at single frequency, Ω_0, for which

$$\left|\frac{L_{ZOH}(e^{j\Omega_0})}{1+L_{ZOH}(e^{j\Omega_0})}\right|=M. \tag{12}$$

The (a,b) pairs inside the allowed region satisfy

$$\left|\frac{L_{ZOH}(e^{j\Omega})}{1+L_{ZOH}(e^{j\Omega})}\right|<M \tag{13}$$

for all Ω, meaning that it is possible to decrease b for a fixed a. The importance of this observation follows from Seron and Goodwin [32] who state: "In general, the process noise spectrum is typically concentrated at low frequencies, while the measurement noise spectrum is typically more significant at high frequencies." Thus, it is possible to preserve low frequency sensitivity and disturbance rejection properties while decreasing the sensor noise effect at the plant input and output. In terms of a and b, this means that the smallest possible b for a fixed a is best, leading to a choice of (a,b) on the boundary (where b is minimal).

The controller design with lowest sensitivity at low frequencies, i.e., with maximum a, is directly available by inspection of Fig. 1. For $M=1.46$ and $K=1$, the values for a controller with lowest sensitivity are $(a,b)=(-20.3\,\text{dB},6.07)$. With an additional 6 dB of plant gain uncertainty, the values are $(a,b)=(-26.4\,\text{dB},6.34)$. The corresponding (A,B) values for general delay time T can be calculated from (a,b) using (7).

The solution for different M values is depicted in Fig. 2. Each curve is the boundary of the allowed (a,b) values for a given M. The corresponding PM and GM (for $K=1$) values indicated are a lower bound, that is, (a,b) values on a curve guarantee at least the PM and GM indicated. Assuming a plant gain uncertainty of $[1,K]$, the question is how to find the range of (a,b) values that satisfies the margin conditions and gain uncertainty. The answer is the range between the lower bound of the (a,b) region and its upper bound shifted down by K (dB units). The maximum a and its associated b can then be extracted from Fig. 2, as explained in Fig. 1 for the case of $M=1.46$ and $K=2$.

3.2 Approximate Solution. It is possible to simplify the equations for the exact solution by replacing the delay T in (6) by a ZOH with sampling of duration T_s. The ZOH of the open-loop transfer function (6) is then

$$L_{ZOH}(z)=\frac{AT_s^2(z+1)}{2(z-1)^2}+\frac{ABT_s}{z-1}. \tag{14}$$

Using the change of variables,

$$a=AT_s^2,\quad b=\frac{B}{T_s},\quad \Omega=\omega T_s,\quad Z=e^{j\Omega}, \tag{15}$$

(14) can be written

[1]The equations were derived using Matlab's Symbolic Mathematics Toolbox from MathWorks, Inc., Natick, MA.

$$L_{ZOH}(Z)=\frac{a(Z+1)}{2(Z-1)^2}+\frac{ab}{Z-1}. \quad (16)$$

The allowed (a,b) values are those for which the closed-loop system is stable and the margin specification (1) is satisfied. As a first problem, consider the special case of no gain uncertainty. Substituting (16) into (1) with $K=1$ gives

$$\left|\frac{a(Z+1)+2ab(Z-1)}{2(Z-1)^2+a(Z+1)+2ab(Z-1)}\right|\leqslant M, \quad \forall|Z|=1, \quad (17)$$

which can be written using the bilinear transformation $Z=(1+j\Omega)/(1-j\Omega)$ as

$$\left|\frac{a(1+2b\Omega^2)+ja(2b-1)\Omega}{a+(2ab-4)\Omega^2+ja(2b-1)\Omega}\right|\leqslant M, \quad \forall\Omega\geqslant 0. \quad (18)$$

Inequality (18) is satisfied, if and only if, for all $\Omega\geqslant 0$,

$$[-4a^2b^2+M^2(2ab-4)^2]\Omega^4+[M^2(4a^2b^2-8a+a^2)-a^2-4a^2b^2]\Omega^2+[a^2(M^2-1)]\geqslant 0. \quad (19)$$

The (a,b) pairs that solve inequality (19) must be such that the coefficient of Ω^4 is not negative, that is,

$$-4a^2b^2+M^2(2ab-4)^2\geqslant 0, \quad (20)$$

which is a quadratic inequality in terms of ab with the solution of either $ab<2M/(M+1)$ or $ab>2M/(M+1)$. Since $M>1$, satisfaction of the second inequality also guarantees $ab>2$ for which it can be shown that (16) is closed-loop unstable. Thus, the second inequality is inadmissible, and it follows from the first inequality that the condition for a non-negative coefficient of Ω^4 is

$$a\leqslant\frac{2M}{b(M+1)}. \quad (21)$$

Two options exist for the coefficient of Ω^2:

1. The coefficient of Ω^2 is positive. This corresponds to

$$\frac{8M^2}{(M^2-1)(1+4b^2)}\leqslant a \quad (22)$$

The coefficient of Ω^2 is negative. In this case, the two solutions for Ω^2 in the quadratic equality (19) must not be real, which gives

$$(4b^2-1)^2a^2+16\frac{M^2(2b-1)^2}{1-M^2}a+64\frac{M^2}{(1-M^2)^2}\leqslant 0 \quad (23)$$

Since $M>1$, the coefficient of Ω^0 is always positive.

Based on the above observations, the general solution of inequality (19) can be calculated, and is presented in Fig. 3, where each curve is the boundary of the allowed (a,b) values for a given M. The graph ordinate axis is normalized by the factor $M/(M+1)$ based on inequality (21) to achieve a common top curve. As before, the range of (a,b) values which satisfy the margin conditions and gain uncertainty $[1,K]$ is the range between the lower bound of the (a,b) region and its upper bound shifted down by K (dB units).

3.2.1 Example. Consider a problem in which it is required that $M=1.46$ and $K=3.16$ (10 dB), corresponding to at least 40 deg phase margin and at least 14.5 dB gain margin. The curve marked $M=1.46$ in Fig. 3 is selected which corresponds to PM=40 deg or greater and GM=4.5 dB or greater for $K=1$. We seek a b value for which the two boundary values of a differ by 10 dB (to achieve 14.5 dB total). This happens at $b=4.1$ where the two $a(M+1)/M$ values from the graph are −6.3 and −16.3 dB ($a=0.092$ and $a=0.29$, respectively). The Nichols plot for $b=4.1$ and $a=0.092$ (where the lowest a is taken in order to handle the gain uncertainty) is shown in Fig. 4 for $T_s=0.001$ s, corresponding to $A=0.092/T_s^2$ and $B=4.1T_s$ in (14). Note that the gain and phase margin conditions are equivalent to requiring that the plot of the open-loop transfer function does not enter the shaded region encircling the point (0 dB, −180 deg).

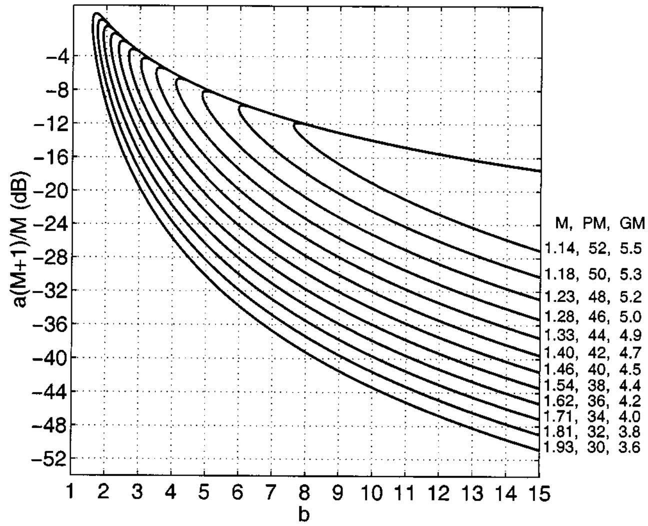

Fig. 3 Boundary curves of (a,b) values that satisfy (1) with $K=1$ where L is replaced by $L(Z)$ of (16). Marked on the right of each curve is its M value and corresponding lower bound of phase margin (PM) and gain margin (GM in dB) for $K=1$, according to (3) and (4).

3.2.2 Explicit Expressions for Lowest Sensitivity Controller. It is possible to determine the parameters of the controller with the lowest sensitivity at low frequencies. Since $1/a$ is proportional to the sensitivity at low frequencies, the solution whose gain, a, is maximum is desired. The maximum a is dictated by (21) and either condition (22) or (23). From (21) and (22), a is maximum when

$$a=\frac{2M}{b(M+1)} \quad \text{and} \quad b=\frac{M+\sqrt{2M-1}}{2(M-1)}. \quad (24)$$

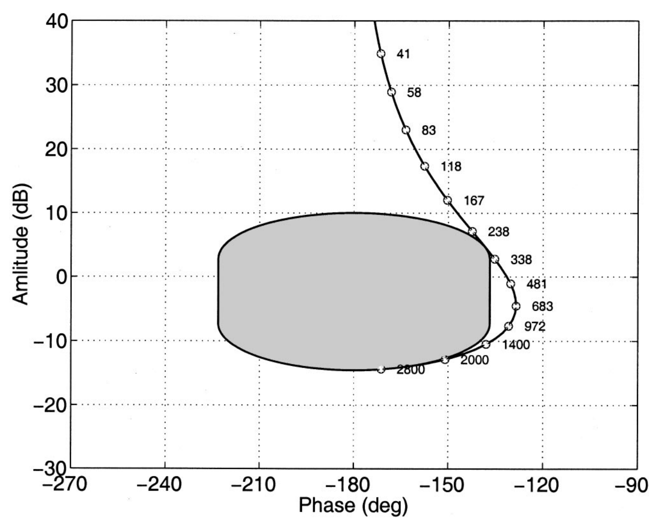

Fig. 4 Nichols plot for $M=1.46$ and $K=3.16$, corresponding to at least 40 deg phase margin and at least 14.5 dB gain margin (4.5 dB due to M and 10 dB due to plant gain uncertainty). Frequencies are marked in rad/s for chosen $T_s=0.001$ s. The open-loop transfer function must not enter the shaded region in order to satisfy the gain and phase margin constraints.

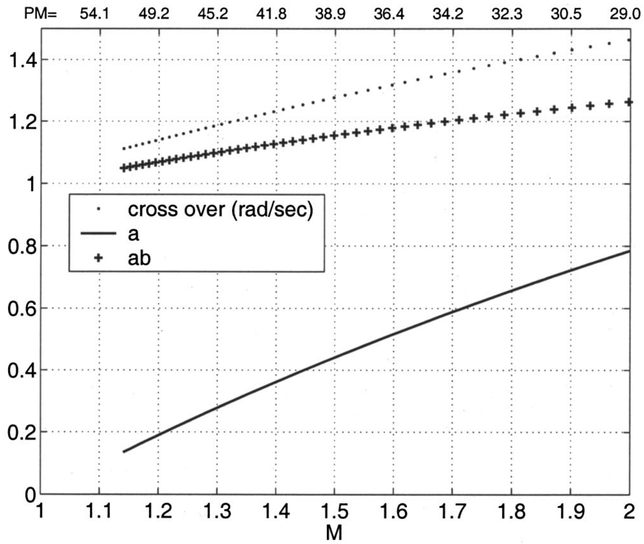

Fig. 5 Maximum *a*, *ab* and crossover frequency (rad/s) versus *M* and its associated guaranteed phase margin (deg) (indicated at top) for the approximate solution.

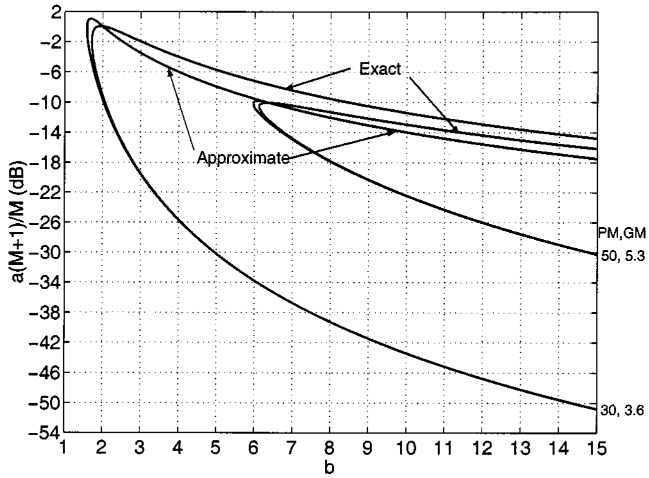

Fig. 6 Boundary curves of (a,b) values showing comparison between exact and approximate solutions for $M=1.93$ (phase margin, *PM*, of at least 30 deg) and for $M=1.18$ (PM of at least 50 deg). Gain margin, GM, indicated in dB.

For (21) and (23), the maximum a is derived as follows. Use the equality of (23) as an extremum problem for a, i.e., calculate the derivative of a with respect to b and then equate to zero for maximum a. The extremum pair of (a,b) then must be bounded by

$$ab(2b+1)\leqslant \frac{4M^2}{M^2-1}. \qquad (25)$$

However, the pair (a,b) of (24) satisfies (25), and hence the equality of (25) will dictate the maximum a. Substituting the equality of (25) in the equality of (23) gives the following cubic equation in terms of b for maximum a,

$$8(M^2-1)b^3-4(3M^2+1)b^2+6M^2b-M^2=0. \qquad (26)$$

Of the three solutions of (26), the only admissible solution is the one which is positive and for which the closed-loop system is stable. Substituting this solution into the equality of (25) gives the maximum of a.

Figure 5 depicts the maximum a and its ab value versus M and the corresponding lower bound of the phase margin, according to (4). The crossover frequency, which is also depicted in Fig. 5, is calculated from $|L_{ZOH}(e^{j\Omega})|=1$. The result is the following quadratic equation in $\cos(\Omega)$,

$$-16\cos^2(\Omega)+(2a^2-8a^2b^2+32)\cos(\Omega)+(2a^2+8a^2b^2-16)=0, \qquad (27)$$

which can be solved for Ω.

From Fig. 5, when M decreases from $M=2$ to $M=1.3$, corresponding to an increase in the phase margin from at least 29 deg to at least 45 deg, the crossover frequency decreases by a factor of 1.21, ab decreases by 1.14, and the sensitivity at low frequencies, $1/a$, increases by 2.6. Since, in general, the sensor noise is proportional to ab [32], the noise will not increase much when the phase margin is decreased. However, the loop gain at low frequencies which is proportional to a changes significantly. Since the loop gain is responsible for disturbance rejection and sensitivity, the conclusion is that the dominant price of the phase margin increase is greater disturbance rejection and sensitivity.

3.3 Comparison of Exact and Approximate Solutions. A reasonable approximation to the exact (a,b) region can be calculated from the approximate equations where $T_s=\alpha T$ and $\alpha=2$. The reason for using $\alpha=2$ is that the phase of $1/s$ and $1/s^2$ is the same as the phase of its ZOH equivalent form, respectively. It is, therefore, expected that the two regions almost overlap. The (a,b) region for the exact problem can therefore be compared to the region $(a\alpha^2,b/\alpha)$. Figure 6 is a comparison of the exact and approximate (a,b) regions for $M=1.66$ (at least 30 deg phase margins) and $M=1.18$ (at least 50 deg phase margin). The approximate solution region is internal to the exact solution region, except for a very small region near maximum a, which decreases with the phase margin and gain margin. However, when using this added region for the exact case, the violation of the margin conditions is small. By conducting frequency domain simulations for $M=6.0$, 4.4, and 3.3 (corresponding to phase margins of at least 30, 35, and 40 deg, respectively), it can be shown that condition (1) is at most violated by less than 0.78, 0.44, and 0.23 dB, respectively, and decreases further for larger phase margins.

3.4 Extension to Sensitivity Requirements. Replacing the complementary sensitivity (1) by the sensitivity margin requirements,

$$\left|\frac{1}{1+kL(s)}\right|\leqslant M, \quad \text{for } s=j\omega, \ \forall\omega>0, \ k\in[1,K], \qquad (28)$$

it can be shown that $L=L_0$ satisfies (1) if and only if $L=[(M^2-1)/M^2]L_0$ satisfies (28). This leads to the following corollary: Let (a,b) be a pair that solves the problem stated in Sec. 2. Then the pair,

$$\left(\frac{M^2a}{M^2-1},b\right),$$

solves the same problem where (1) is replaced by (28).

4 Conclusions

The paper studies the design of nonminimum plants of the pure delay type that satisfy simultaneously robustness to gain uncertainty and an upper bound on the (complementary) sensitivity. The paper proposes a control structure which involves a PI controller in series with the inverse of the minimum phase part of the plant. Using this form, all stabilizing controllers, including the one with the smallest sensitivity, can be determined. In addition to explicit expressions, a design plot is introduced which enables the selection of the PI controller which guarantees the gain and phase margin specifications over the plant gain uncertainty. Moreover, the plot can be used to identify the controller with lowest sensitivity, and to uncover tradeoff issues associated with gain margin, phase margin, sensitivity, and high-frequency sensor noise at the plant input. Expressions and design plots are provided for both the exact solution and a simplified approximate solution.

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